$$m_1 r^{-m-1}\{-nd+[r-\tfrac{3}{2}(m+1)r^{-1}d^2](\cos\alpha+\cos2\alpha+\cos3\alpha+\ \ldots\ldots\cos n\alpha)$$
$$+(m+1)d(\cos^2\alpha+\cos^2 2\alpha+\cos^2 3\alpha+\ldots\ldots\cos n\alpha)$$
$$+\tfrac{1}{2}[(m+1)(m+3)]r^{-1}d^2(\cos^3\alpha+\cos^3 2\alpha+\cos^3 3\alpha+\ldots\ldots\cos^3 n\alpha)\}.$$

By trigonometry,

$$\cos\alpha+\cos2\alpha+\ldots\ldots\cos n\alpha=\frac{\cos\frac{1}{2}(n+1)\alpha.\sin\frac{1}{2}n\alpha}{\sin\frac{1}{2}\alpha};$$

$$\cos^2\alpha+\cos^2 2\alpha+\ldots\ldots\cos^2 n\alpha=\tfrac{1}{2}\{n+\frac{\cos(n+1)\alpha.\sin n\alpha}{\sin\alpha}\};$$

$$\cos^3\alpha+\cos^3 2\alpha+\ldots\ldots\cos^3 n\alpha=\frac{\cos\frac{1}{2}(n+3)\alpha.\sin\frac{3}{2}n\alpha}{4\sin\frac{3}{2}\alpha}+\frac{3\cos\frac{1}{2}(n+1)\alpha.\sin\frac{1}{2}n\alpha}{4\sin\frac{1}{2}\alpha}.$$

The value of these series when $n\alpha=2\pi$ are 0, $n/2$, and 0, respectively.

Hence the expression for the approximate value of the resultant force reduces to

$$m_1 r^{-m-1}[-nd+(\tfrac{1}{2}n)(m+1)d],\ \text{or}\ \frac{m_1 n(m-1)}{2r^{m+1}}d.$$

Also solved by *G. B. M. ZERR.*

PROBLEMS FOR SOLUTION.

ARITHMETIC.

95. Proposed by WALTER HUGH DRANE, A. M., Professor of Mathematics, Jefferson Military College, Washington, Miss.

Solve by arithmetic, if possible.

A man sold a house for $7500 and gained a certain per cent. on the cost. If the cost had been 16⅔% less, his gain would have been 25% greater. Find the cost of the house.

96. Proposed by RAYMOND SMITH, Tiffin, Ohio.

How many acres in a square field whose diagonal is 10 rods longer than the side ?

*** Solutions of these problems should be sent to B. F. Finkel, not later than April 10.

ALGEBRA.

85. Proposed by J. M. COLAW, A. M., Monterey, Va.

Sum the infinite series

$$\frac{1}{1^2.2^2.3^2.4^2.5^2}+\frac{1}{2^2.3^2.4^2.5^2.6^2}+\frac{1}{3^2.4^2.5^2.6^2.7^2}+,\ \text{etc.}$$

86. Proposed by J. MARCUS BOORMAN, Consultative Mechanician, Counselor at Law, Inventor, Etc., Woodmere, Long Island, N. Y.

Solve $x^2+yz=16\ \ldots\ldots(A)$; $y^2+xz=17\ldots\ldots(B)$; $z^2+xy=22\ldots\ldots(C)$, for all the roots.

[This is Col. Titus' problem—see "Maseres' Tracts," pages 188-276—and is solved by Dr. Wallis in 51 pages, and by Mr. Frend in 38 pages, 8vo., but by the writer in 1 or 2 pages, 4to., or less. J. M. B.]

87. Proposed by the late B. F. BURLESON, Oneida Castle, N. Y.

A starts to travel around a circular island at a given point and travels at the rate of 5 miles in 4 hours. One half hour after A, B starts from a point directly opposite from A and travels in an opposite direction at the rate of 4 miles in 3 hours. One hour afterwards C starts from the same point as A and travels in an opposite direction to A at the rate of 3 miles in 2 hours. One half hour afterwards D starts from the same point as B and travels in an opposite direction to B at the rate of 2 miles in 1 hour. Required the size of the island, and when they will all be together, and how far each will have traveled at the accomplishment of this event.

*** Solutions of these problems should be sent to J. M. Colaw, not later than June 10.

GEOMETRY.

94. Proposed by EDMOND FISH, Hillsboro, Ill.

A tower $AB=a$, is surmounted by a flag pole $BC=b$. A point D is so taken in a line perpendicular to the foot of the tower that angle BDC is a maximum. Prove that AD is a mean proportional between AC and AB.

95. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy in Ohio State University, Athens, Ohio.

At each point of a parabola is described the rectangular hyperbola of four-pointic contact; prove that the locus of the center of the hyperbola is an equal parabola.

96. Proposed by W. F. BRADBURRY, A. M., Head Master, Cambridge Latin School, Cambridge, Mass.

Isosceles triangles are constructed externally on the three sides of a triangle as bases, with the angles at the bases each 30°. The triangle formed by joining the remote vertices (the 120° vertices) of these isosceles triangles is equilateral. [Geometric—not Trigonometric—solution.]

*** Solutions of these problems should be sent to B. F. Finkel, not later than June 10.

CALCULUS.

75. Proposed by O. W. ANTHONY, M. Sc., Instructor in Mathematics in Boys' High School, New York City.

Solve the differential equation

$$\frac{d^2y}{dx^2}+n^2y=\frac{6}{x^2}y.$$

76. Proposed by E. B. ESCOTT, Cambridge, Mass.

Solve the partial differential equation

$$q^2r+4pqs+p^2t+p^2q^2(rt-s^2)=a^2.$$

[*Forsyth's Differential Equations*, page 376.]

77. Proposed by T. E. COLE, Columbus, Ohio.

Derive the equation of a point in the pedal of a bicycle as the wheel rolls along on a plane.

*** Solutions of these problems should be sent to J. M. Colaw, not later than June 10.